METHODS OF SOLVING ILL-POSED INVERSE PROBLEMS

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A number of methods of solving inverse heat-conduction problems are analyzed from the point of view of their practical use. Problems of determining discrepancy gradients and obtaining smooth solutions are considered as applied to the method of iteration regularization.

1. At the present time there exist two directions in the study of effects of heat and mass transfer, viz., the active development of methods of numerical modeling on the basis of equations expressing conservation laws, and continual improvement of experimental methods. If, in this connection, one considers the role of inverse heat-conduction problems (IHCP), they are important for both the first and second direction, and inverse problems particularly make it possible to extract quantitative information on the sources of characteristics appearing in the mathematical model of the effect (coefficients of equations, boundary conditions, etc.), and recover necessary data from experimental studies.

Application areas of these methods are thermophysical and heat-technology studies in various fields of science, technology, industry, energetics, machine construction, aviation technology, metallurgy, etc.

Obviously, it can be stated that IHCP methods underwent through their development stages of formulation of primary concepts, statement of problems and their solutions, as well as formulation of basic methods and examples of solving problems. The state-of-the-art in this area is characterized by a set of concepts and methods, which can be uncoupled as initial form of the theory and methodology of inverse problems. Further accumulation of abstract and concrete facts must lead to their union in a general theoretical system, based on uniqueness principles, rigorous mathematical studies of correctly stated problems, and further development of the IHCP algorithm apparatus. At the same time, one must carry out an all-rounded practical investigation of inverse problem methods, and clarification of the most rational applications. It must be stressed that contemporary IHCP methods are oriented toward using computational technology, and only in exceptional cases are "manual" treatments allowed.

2. The concepts of direct and inverse problems always refer to mathematical model effects, separated by causal and consequential characteristics of the process investigated. In the direct problem the "consequence" results from the "cause," while in inverse problems this happens in the opposite direction.

The abstract form of an inverse problem is represented by an operator equation of the first kind

$$Au = f, \quad u \in U, \quad f \in F \tag{1}$$

with a given operator A: U → F and an element f, from which one seeks a solution u.

A characteristic feature of inverse problems is that the operator A has no bounded inverse, i.e., problem (1) is incorrect.

One possibility is the parametrized statement of the inverse problem, for example, when the quantity u(x) is sought in the form

$$u(x) = \sum_{i=1}^{l} a_i \varphi_i(x), \tag{2}$$

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where $\{\varphi_i(x)\}_1^l$ is a system of basis functions and a_i are unknown coefficients.

An example of representation (2) can be the approximation of a solution by B-splines. In this case the inverse problem reduces to determining the vector $\bar{a}=\{a_i\}_1^l$.

One must mention the lasting delusion that in transforming to a parametrized identification form (unlike the more general case of functional identification) the difficulties associated with obtaining a stable solution are eliminated. The use of a similar form in the general case leads to the result that due to its original incorrectness the finite-dimensional analog of the problem is badly conditioned, and also requires the development of special algorithms of finding an approximate solution, stable with respect to small changes in the original data. One method of constructing similar algorithms for the spline approximation can be the natural step regularization. In this case stability of the solution is achieved by a choice of the partitioning step of the region of determining the unknown function, larger than the quantity for which the conditioning of the problem improves so much that the error in the right-hand side has a comparatively small effect on the solution [1-4]. Naturally, this approach has restrictions related to the worsening of results with the necessary coarsening of the computational grid.

A quite general method of solving ill-posed problems is the regularization method of Tikhonov [5, 6], and among the principles of constructing regularized algorithms the most widely spread is the variational principle [5-8], being widely used in IHCP solutions [1, 9]. Also used are other methods of obtaining stable solutions, among which use was found for inverse problems by the already-mentioned method of natural step regularization of approximately analytic and difference-shaped solutions, as well as the iteration regularization method suggested in [10, 11].

All these approaches were investigated by us in sufficient detail, as applied to solving both linear and nonlinear IHCP, and were compared with each other in various practical situations. As the studies performed have shown in the present case, the conditions of rational practical application of methods of solving IHCP can be generalized in the form of Table 1. Five approaches to algorithmization of solutions of IHCP, making it possible to obtain stable results, are summarized in it. These methods are systematically discussed in [1] (the direct approximate-analytic and numerical methods in Chaps. 4 and 5, the iteration regularization method in Chap. 6 and Sec. 7.6, and algebraic and numerical methods, regularized according to variational principles, in Chap. 7).*

From analyzing this table one can draw the conclusion that among the approaches considered to solving inverse problems the most universal is the method of iteration regularization, based on gradient algorithms. Due to a number of obvious qualities this method has been applied by us very widely, so that, finally, it does not exclude the application of other methods, which can be more rational in other cases.

3. We discuss briefly the essence of the given approach and several results obtained in developing this direction recently.

Instead of (1) we consider the extremal analog, more precisely, the minimization problem of the norm of the deviation of the left-hand side of the equation from the right-hand side in the metric of the space F:

$$J(u) = \|Au - f\|_F^2, \tag{3}$$

and as F we take the space of square integrable L_2-functions. This approach does not render the problem correct, but, as it turns out, one can construct an effective regularizing algorithm of its solution, based on an iteration process

$$u^{j+1} = F_A(u^j, \delta f), \quad j = 0, 1, \ldots,$$

when the iteration number is considered as a regularization parameter (here δf is the error in the given element f).

*In the present paper we are not concerned with other algorithms and methods of solving IHCP, which can be found, in particular, in [12-17].

TABLE 1. Conditions of Rational Application of Methods of Solving IHCP

Condition	I	II	III	IV	V
Feature of IHCP					
Linear with constant thermophysical characteristics (TFC)	Yes	Yes	Yes	Yes	Yes
Nonlinear or linear with varying TFC	No	Yes	Yes	No	Yes
Homogeneous equation of thermal conductivity	Yes	Yes	Yes	Yes	Yes
Generalized equation of thermal conductivity	No	Yes	Yes	No	Yes
Immobile body boundaries	Yes	Yes	Yes	Yes	Yes
Mobile body boundaries	In the one-dimensional case	Yes	Yes	In the one-dimensional case	Yes
Fixed thermal detector	Yes	Yes	Yes	Yes	Yes
Mobile thermal detector	No	Yes	Yes	No	Yes
One-dimensional	Yes	Yes	Yes	Yes	Yes
Two-dimensional	For regions of simple shape	Yes	Yes	For regions of simple shape	No
Overdetermined	No	No	Yes	No	No
General applicability conditions					
Determination of thermal loads	Yes	Yes	Yes	Yes	Yes
Calculation of temperature fields	No	Yes	Yes	No	Yes
Heat exchange (TP) slowly varying in time	Yes	Yes	Yes	Yes	Yes
Quickly varying and short-lived TP	No	No	Yes	Yes	Yes
Low-temperature TP	Yes	Yes	Yes	Yes	Yes
High-temperature TP of substantially varying intensity	No	Yes	Yes	No	Yes
Low thermal depth of thermal detector	Yes	Yes	Yes	Yes	Yes
High thermal depth of thermal detector	No	No	Yes	Yes	Yes
Constructive metallic elements	Yes	Yes	Yes	Yes	Yes
Thermal shielding and isolation	No	For a material with high thermal cond.	Yes	Yes	Yes
Conditions of practical realization					
Complication of algorithm	Simple	Average complication		Substantial complication	Complicated
Cost of machine time	Small	Small, average	Average		Large
Calculated in a real time scale	Possible				Impossible
Stability restrictions on the magnitude of calculated steps	Quite rigid	Rigid	Absent		
Preliminary smoothing of input data for high error levels	Necessary		Not necessary		

TABLE 1 (continued)

Condition	I	II	III	IV	V
Obtaining a required degree of smoothness of results	Impossible			Possible	
Account of quantitative a priori information on solutions	Impossible		Possible	Difficultly realizable	
Conditions of computer realization					
Computer	Yes	Yes	Yes	Yes	Yes
Hybrid computer system (HCS)	No	For implicit difference scheme	Yes	Yes	Yes

Note: I) direct approximate-analytic method; II) direct numerical method; III) method of iteration regularization; IV) regularized algebraic method; V) regularized numerical method.

The methods of steepest descents, minimal discrepancies, and associated gradients were investigated as iteration algorithms. It has been established that if the iteration number is chosen from the discrepancy criterion $\|Au-f\|^2 \simeq \delta^2$, then these methods make it possible to obtain stable and quite accurate approximations to the required solution of an incorrect problem. Numerical modeling, based on solving numerous model examples for various IHCP types, as well as problems of smoothing and differentiating functions given with errors have shown the correctness and effectiveness of this approach both for linear and nonlinear problems. As applied to linear inverse problems, i.e., to the case of a linear operator A, the given method is rigorously justified. Theorems on conditional regularization of iteration algorithms and stability of approximations in gradient methods in using discrepancy criteria were given in [18, 19]. Conditions were established in [20], according to which one infers from the validity of the discrepancy criterion for a given iteration algorithm F_A the validity of generalized discrepancy criteria, and generalized discrepancy criteria were also justified for simple iteration and reconstruction methods.

We dwell further on two important problems of applying the method of iteration regularization: the determination of the J'_u-gradient of the functional (3), and the development of algorithms taking into account the smoothness of the defining functions (vector-functions).

4. A numerical method of calculating a gradient, based on the difference approximation of partial derivatives with respect to separate components, is widely used in computational practice in solving correct problems (it is assumed that a parametrization procedure of the unknown functions is carried out). This approach is not only accompanied by substantial expenditure of computer time, but is also undesirable for the reason of large errors. Also unsuitable for calculating the gradient is another well-known method, based on sensitivity functions. This is related to the immense computational bulk, since the method assumes that one has solved a boundary-value problem (for example, the problem for the heat-conduction equation), whose dimensionality is that of the vector of unknown parameters.

As studies have shown, more effective algorithms of determining the discrepancy gradient can be obtained if the following exact representation is used to calculate J'_u:

$$J'_u = 2(A')^*(Au - f),$$

where A' is the Fréchet derivative of the operator A, and $(A')^*$ is the operator conjugate to A' (if Eq. (1) is linear, then $(A')^* = A^*$). Two cases are possible here:

1) when the operator A is a linear integral operator, and one easily and explicitly constructs for it the conjugate integral operator (in particular, a similar situation arises in solving linear IHCP boundary-value problems [1]);

2) when to find $(A')^*$ one uses the solution of the boundary-value problem conjugate to the problem for the increment of the unknown function (vector-function); the method of deriving the conjugate problems can be justified by considering the necessary conditions of

extremum of the increment functional (3) in the form of vanishing of the total variation of the extended functional [21].

The methods mentioned of finding the gradient possess the following important features:

universality, they are valid for solving inverse problems of any type and in various contexts, not only for a parametric form of identification, but also for functional identification;

high accuracy, in calculating J'_u there are practically only approximation errors, which can be made negligibly small;

economy, for example, if J'_u is found in terms of the solution of the conjugate problem, then independently of whether functional or parametrized identification is used, and independently of the dimensionality of the vector sought (or vector-function), at each iteration it is required to solve the conjugate problem only once in calculating the gradient.

We note that the second method is applicable both to linear and nonlinear IHCP. The conjugate problems for various boundaries and coefficients of inverse heat-conduction problems and inverse heat-conduction problems in technological systems, as well as the corresponding expressions for the gradients of the mean-square functionals are given in [1, 10, 11, 21-26].

5. An equation is given below for the gradient of the discrepancy functional for the iteration solution of the new boundary-value inverse heat-conduction problem, in whose statement there is no initial temperature distribution (in a number of given conditions).

In the one-dimensional case we formulate this problem as follows: find the temperature field T(x, τ) in the region $\Omega=\{(x, \tau): 0\leqslant x<d,\ 0\leqslant\tau\leqslant\tau_m\}$ from known boundary conditions of the fourth kind at the point x = d

$$T(d, \tau)=f(\tau), \quad -\lambda\frac{\partial T(d, \tau)}{\partial x}=q(\tau), \tag{4}$$

assuming that the function T(x, τ) satisfies the heat-conduction equation

$$C\frac{\partial T}{\partial \tau}=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right). \tag{5}$$

Stated this way, the problem is called a noncharacteristic Cauchy problem for Eq. (5). If the coefficients C and λ are functions of x, τ and are independent of temperature, the heat-conduction equation can be written in the form

$$\frac{\partial T}{\partial \tau}=a(x, \tau)\frac{\partial^2 T}{\partial x^2}+b(x, \tau)\frac{\partial T}{\partial x}. \tag{6}$$

Here

$$a(x, \tau)=\frac{\lambda}{C}>0, \quad b(x, \tau)=\frac{1}{C}\frac{\partial\lambda}{\partial x}.$$

For classical shapes of problem (6), (4), when the coefficients of the equation are analytic functions of x, τ, the input data f(τ), q(τ) and the field sought T(x, τ) also belong to the class of analytic functions, and the uniqueness of the solution follows from the Kovalevskii theorem. For substantially less restrictive requirements, more precisely when $T(x, \tau)\in C^2$, the Cauchy data $f(\tau), q(\tau)\in C^1$, and the coefficients satisfy the conditions $a(x, \tau)\in C^2$ and b(x, τ) is a bounded function, then the uniqueness of the solution follows from the uniqueness theorem of the noncharacteristic Cauchy problem for the general n-th-order parabolic equation, proved in [27]. In what follows we refer to this result, and assume that Ω is located inside a region for which the uniqueness of the solution of the problem has been established.

The inverse problem (6), (4) on the extension of the temperature field from Cauchy data is reformulated in the following variational form: considering as variational functions the boundary temperature $T_w(\tau)=T(0, \tau)$, $\tau \in [0, \tau_m]$ and an initial distribution $\varphi(x)=T(x, 0)$, $x \in [0, d]$, find by the iteration regularization method a pair (T_w, φ), for which the discrepancy functional

$$J[T_w(\tau), \varphi(x)] = \int_0^{\tau_m} [T(T_w, \varphi, b, \tau) - f(\tau)]^2 d\tau \quad (7)$$

acquires a value equal to the integral error δ^2 of the given function $f(\tau)$. In this case the direct problem of thermal conductivity is solved at each iteration for the known approximations for $T_{wj}(\tau)$ and $\varphi_j(x)$, with the second boundary condition on the heat flux density $q(\tau)$ (assumed in the given case to be a known function). The unknown field $T(x, \tau)$ in the region Ω is calculated from the quantities found $T_w(\tau)$, $\varphi(x)$ and the given $q(\tau)$.

We note that in the formulation suggested the original Cauchy problem reduces to the extreme statement of the boundary-retrospective IHCP (according to the terminology proposed in [1]).

According to the uniqueness conditions of the solution of problem (6), (4) it is necessary to seek, among the functions, $T(x, \tau) \in C^2$; therefore the varying quantities $T_w(\tau)$ and $\varphi(x)$ refer to this class. Moreover, we assume that they belong to the space of W_2^3-functions, having generalized derivatives up to third order, and being square-integrable (it is well-known that $W_2^3 \subset C^2$). Thus, to apply the gradient minimization method with account of the restrictions it is necessary to find the gradient of the functional (7) with respect to the vector-function $u=[T_w(\tau), \varphi(x)]$, where $T_w \in W_2^3[0, \tau_m]$, $\varphi \in W_2^3[0, d]$. In solving this problem it is necessary to have agreement between the unknown functions $T_w(\tau)$, $\varphi(x)$ and the given boundary conditions, in particular, it is necessary to satisfy the equalities

$$T_w(0) = \varphi(0), \quad \varphi(d) = f(0), \quad -\lambda\varphi'(d) = q(0).$$

In Sec. 6 below, a procedure is suggested for determining the gradient in the W_2^k-space from the gradient known in the L_2-space. Consequently, it is initially necessary to solve the basic problem, that is, to construct the gradient J'_u for $T_w \in L_2[0, \tau_m]$, $\varphi \in L_2[0, d]$. For this purpose we use the method of the conjugate boundary-value problem.

Not dwelling on details, we provide the final results. The conjugate problem sought is

$$-\frac{\partial\psi}{\partial\tau} = \frac{\partial^2(a\psi)}{\partial x^2} - \frac{\partial(b\psi)}{\partial x}, \quad x \in (0, d), \quad \tau \in [0, \tau_m);$$

$$\psi(x, \tau_m) = 0; \quad \psi(0, \tau) = 0;$$

$$-b(d, \tau)\psi(d, \tau) + \frac{\partial}{\partial x}[a(d, \tau)\psi(d, \tau)] = 2[T(d, \tau) - f(\tau)].$$

The gradient of the functional is expressed in terms of the function ψ as follows:

$$J'_\varphi = \psi(x, 0), \quad J'_{T_w} = -b(0, \tau)\psi(0, \tau) + \frac{\partial}{\partial x}[a(0, \tau)\psi(0, \tau)].$$

After calculating from these data the gradient in the W_2^3 space with account of the self-consistency mentioned it is possible to organize in parallel two iteration processes for finding the quantities $T_w \in W_2^3[0, \tau_m]$, $\varphi \in W_2^3[0, d]$.

6. It is well known that the quality of solution of an ill-posed problem can be improved substantially if one includes in the algorithm the a priori given smoothness of the

dominant functions. An approach was suggested in [1, 28] and further developed in [29] toward finding smooth solutions within the method of iteration regularization. According to this approach, to solve problem (1) by the gradient method one constructs an iteration sequence, in which the direction of descent is selected in the original space $U = L_2$, in such a manner that the approximations obtained belong to the class W_2^k. A new form of finding smooth solutions is suggested below, when the iteration sequence is obtained directly in the space $U = W_2^k$.

We assert that the required function u(x) belongs to the Sobolev space of $W_2^k[a, b]$-functions, square integrable along with their derivatives up to the k-th order, where the derivatives are understood in the sense of generalized functions. It is well known that in this case the function u(x) is continuously differentiable (k − 1) times on the [a, b] segment, and has almost everywhere in it an ordinary k-th derivative, while the derivative $u^{(k-1)}(x)$ is absolutely continuous on [a, b].

Let u(x) acquire a small increment $\theta(x) \in W_2^k$. Then the linear part of the corresponding increment of the functional J is expressed in terms of the scalar derivative in the W_2^k space of the element θ and of the gradient $J'_{W_2^k}$ in this space

$$\Delta J = (\theta,\ J'_{W_2^k})_{W_2^k}, \tag{8}$$

where

$$(\theta,\ J'_{W_2^k})_{W_2^k} = \sum_{n=0}^{k} \int_a^b r_n \frac{d^n J'_{W_2^k}}{dx^n} \frac{d^n \theta}{dx^n} dx.$$

The quantities $r_n = r_n(x)$ are given nonnegative continuous functions, playing the role of weights (in particular, r_n can be constant), while r_0, $r_k > 0$.

Sequentially applying integration by parts, the scalar product (8) can be transformed to a form in which the function θ(x) under the integral sign is free of differentiation (it is assumed here that $J'_{W_2^k}(x)$ and $r_n(x)$ possess the required continuous derivatives):

$$(\theta,\ J'_{W_2^k})_{W_2^k} = \int_a^b \theta(x) \sum_{n=0}^{k} (-1)^n \frac{d^n}{dx^n}\left(r_n \frac{d^n J'_{W_2^k}}{dx^n}\right) dx + \sum_{n=1}^{k} (-1)^{n+1} \theta^{(n-1)}(x) \sum_{i=n}^{k} (-1)^{i+1} \frac{d^{i-n}}{dx^{i-n}}\left(r_i \frac{d^i J'_{W_2^k}}{dx^i}\right)\Bigg|_{x=a}^{x=b}. \tag{9}$$

We introduce the linear differentiation operator $L^{2k} = \sum_{n=0}^{k} (-1)^n \frac{d^n}{dx^n}\left(r_n \frac{d^n}{dx^n}\right)$ and for simplicity we assign the following boundary conditions on the segment [a, b]:

$$\sum_{i=n}^{k} (-1)^{i+1} \frac{d^{i-n}}{dx^{i-n}}\left(r_i \frac{d^i J'_{W_2^k}}{dx^i}\right)\Bigg|_{x=a;\,b} = 0, \quad n = \overline{1, k}. \tag{10}$$

We then obtain from (9)

$$(\theta,\ J'_{W_2^k})_{W_2^k} = (\theta,\ L^{2k} J'_{W_2^k})_{L_2}.$$

Taking (8) into account, we arrive as a result at the differential equation

$$\sum_{n=0}^{k} (-1)^n \frac{d^n}{dx^n}\left(r_n \frac{d^n J'_{W_2^k}}{dx^n}\right) = J'_{L_2}(x), \quad x \in (a,\ b),$$

which is solved with account of the boundary conditions (10).

Thus, a convenient form was obtained for transition from the known gradient of the functional in the L_2 space to the gradient of this functional in the W_2^k space. Knowing $J'_{W_2^k}$ one can construct an approximation process by the method of iteration regularization, providing a result u(x) with a certain degree of smoothness. For example, for the steepest descent method the iteration process

$$u^{j+1} = u^j - \beta_j J'^j_{W_2^k}, \quad \beta_j = \frac{\|J'^j_{W_2^k}\|^2_{W_2^k}}{\|AJ'^j_{W_2^k}\|^2_{L_2}} \tag{11}$$

with residual discrepancies (i.e., by the condition $\|Au^{j*} - f_\delta\|_{L_2} \simeq \delta$) gives the result $u^{j*} \xrightarrow[W_2^k]{} \bar{u}$ for $\delta \to 0$, where $\bar{u} \in W_2^k$ is the solution of problem (1) for exact data. In this case the initial approximation must be selected from the class of functions of corresponding smoothness: $u^0(x) \in W_2^p[a, b]$, $p \geqslant k$, in particular, one can put $u^0(x) = 0$.

It is important to note that if the iteration method of regularization is used to determine some vector-function (for example, when the density of thermal fluxes is sought on two boundaries of the body, or some coefficients in the heat-conduction equation are required, or boundary and initial conditions, as in the problem considered in Sec. 5), the descent step is advisably considered as a vector, having a dimensionality corresponding to the number of dominant quantities [30]. In this approach the accuracy in the solution of the problem is substantially improved.

7. In conclusion, we note that the method of iteration regularization on the basis of gradient algorithms possesses invariance properties with respect to solving inverse problems of various types, boundary value, coefficient, retrospective and geometric inverse heat-conduction problems, inverse problems of heat and mass transfer, inverse heat conduction problems in technological systems, combined inverse problems, inverse problems in redefined statement, as well as optimization problems of project parameters of heat technology systems. Corresponding algorithms are suitable for creating an applied mathematical tool of handling experimental data in automated systems of scientific studies and systems of automated projects.

LITERATURE CITED

1. O. M. Alifanov, Identification of Heat-Exchange Processes of Aircraft (Introduction to the Theory of Inverse Heat-Conduction Problems) [in Russian], Mashinostroenie, Moscow (1979).
2. N. V. Shumakov, The Sequential Interval Method in Thermometry of Nonstationary Processes [in Russian], Atomizdat, Moscow (1979).
3. N. I. Nikitenko, Studies of Heat and Mass Transfer Processes by the Grid Method [in Russian], Naukova Dumka, Kiev (1978).
4. V. I. Zhuk and A. S. Golosov, "Engineering methods of determining thermal boundary conditions from temperature measurement data," Inzh.-Fiz. Zh., 29, 45-50 (1975).
5. A. N. Tikhonov, "Solution of ill-posed problems and the regularization method," Dokl. Akad. Nauk SSSR, 151, 501-504 (1963).
6. A. N. Tikhonov and V. Ya. Arsenin, Solutions of Ill-Posed Problems, Halsted Press (1977).
7. A. N. Tikhonov, "Nonlinear equations of the first kind," Dokl. Akad. Nauk SSSR, 161, 1023-1026 (1965).
8. V. A. Morozov, "Regularization of ill-posed problems and choice of regularization parameters," Zh. Vychisl. Mat. Mat. Fiz., 8, 295-309 (1968).
9. A. N. Tikhonov and V. B. Glasko, "Methods of determining the temperature of a body surface," Zh. Vychisl. Mat. Mat. Fiz., 7, 910-914 (1967).
10. O. M. Alifanov, "Solution of inverse heat-conduction problems by iteration methods," Inzh.-Fiz. Zh., 26, 682-689 (1974).
11. O. M. Alifanov, "Determination of thermal loads from solutions of nonlinear inverse problems," Teplofiz. Vys. Temp., 15, 598-605 (1977).
12. L. A. Kozdoba and P. G. Krukovskii, Solution Methods of Inverse Problems of Heat Transfer [in Russian], Naukova Dumka, Kiev (1982).
13. Yu. M. Matsevityi and A. V. Multanovskii, Identification in Problems of Heat Conductivity [in Russian], Naukova Dumka, Kiev (1982).
14. K. G. Omel'chenko and M. V. Volodina, "Analysis of inverse heat-conduction problems by self-similar solutions," Inzh.-Fiz. Zh., 29, 63-66 (1975).
15. D. F. Simbirskii, Temperature Diagnostics of Engines [in Russian], Tekhnika, Kiev (1976).
16. A. G. Temkin, Inverse Methods of Heat Conduction [in Russian], Énergiya, Moscow—Leningrad (1973).

17. V. M. Yudin, "Heat distribution in glass plates," Tr. TsAGI, No. 1267, Moscow (1970).
18. O. M. Alifanov and S. V. Rumyantsev, "Stability of iteration methods of solving linear ill-posed problems," Dokl. Akad. Nauk SSSR, 248, 1289-1291 (1979).
19. O. M. Alifanov and S. V. Rumyantsev, "Regularization of iteration algorithms for solving inverse heat-conduction problems," Inzh.-Fiz. Zh., 39, 253-258 (1980).
20. S. V. Rumyantsev, "Regularizing gradient algorithms and their application to handling and interpretation of data of physical experiments," Author's Abstract of Candidate's Dissertation, Moscow (1981).
21. O. M. Alifanov and V. V. Mikhailov, "Solution of nonlinear inverse heat-conduction problems by an iteration method," Inzh.-Fiz. Zh., 35, 1124-1129 (1978).
22. O. M. Alifanov, "Gradient methods of reconstructing the thermal regime boundary," in: Heat and Mass Transfer [in Russian], VI, Vol. 9, Minsk (1976), pp. 85-93.
23. O. M. Alifanov, E. A. Artyukhin, and S. V. Rumyantsev, "Solution of boundary and coefficient inverse heat-conduction problems by iteration methods," ibid., pp. 106-112.
24. E. A. Artyukhin, "Reconstruction of the heat-conduction coefficient from solutions of nonlinear inverse problems," Inzh.-Fiz. Zh., 41, 587-592 (1981).
25. E. A. Artyukhin, "Reconstruction of the temperature dependence of the heat-conduction coefficient from solutions of inverse problems," Teplofiz. Vys. Temp., 19, 963-967 (1981).
26. O. M. Alifanov, "Identification of heat- and mass-transfer processes by inverse problem methods," in: Contemporary Experimental Methods of Heat and Mass Transfer Processes [in Russian], Proc. Int. Summer School, Pt. 2, Minsk (1981), pp. 133-147.
27. M. M. Lavrent'ev, V. G. Romanov, and S. P. Shishatskii, Problems in Mathematical Physics and Analysis [in Russian], Nauka, Moscow (1980).
28. O. M. Alifanov and S. V. Rumyantsev, "A method of solving ill-posed problems," Inzh.-Fiz. Zh., 34, 328-331 (1978).
29. E. A. Artyukhin and S. V. Rumyantsev, "A gradient method of finding smooth solutions of boundary-value inverse heat-conduction problems," Inzh.-Fiz. Zh., 39, 259-263 (1980).
30. E. A. Artyukhin and S. V. Rumyantsev, "Optimal choice of descent steps in gradient methods of solving inverse heat-conduction problems," Inzh.-Fiz. Zh., 39, 264-269 (1980).

SOLUTION OF THE TWO-DIMENSIONAL INVERSE HEAT-CONDUCTION PROBLEM IN A CYLINDRICAL COORDINATE SYSTEM

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The two-dimensional inverse heat-conduction problem is considered. An algorithm of the solution and the results of a trial computation are presented.

Modern thermophysical investigation methods, thermal design, and experimental checkout of thermally stressed systems utilize the principles of inverse problems extensively, which have been recommended well in recent years. The high efficiency of methods to investigate heat-transfer processes which are based on the solution of inverse problems, especially in combination with the automated collection and processing of results, resulted in the development of inverse problems in an independent scientific aspect [1].

Different formulations of inverse heat-conduction problems (IHCP) exist at this time. Depending on the purpose, linear and nonlinear IHCP are utilized. Here one-dimensional heat-conduction models are mainly considered.

The selection of the one-dimensional models is based on those cases when a hypothesis on one-dimensional heating can be taken. This hypothesis is valid for many heat-protection

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